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Published in:
Journal of Economics

Publication date:
1995

Citation for published version (APA):
Bovenberg, A. L., van Aarle, B., & Raith, M. (1995). Monetary and fiscal policy interaction and government debt stabilization. *Journal of Economics*, *62*(2), 111-140.

Vol. 62 (1995), No. 2, pp. 111–140
Journal of Economics
Zeitschrift für Nationalökonomie

Monetary and Fiscal Policy Interaction and Government Debt Stabilization

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Received January 4, 1995; revised version received July 18, 1995

Employing differential games, this paper models the strategic interaction between monetary authorities who control monetization and fiscal authorities who control primary fiscal deficits. We analytically compute and interpret the cooperative and noncooperative Nash open-loop equilibria. Furthermore, we reinterpret unpleasant monetarist arithmetic and analyze the impact of a more conservative central bank. Finally, to explore the consequences of a more independent central bank, we analyze Stackelberg open-loop equilibria.

1 Introduction

During the 1980s, many countries experienced substantial increases in government indebtedness. Therefore, government debt stabilization has become a prominent policy issue. Recently, the OECD (Leibfritz et al., 1994) surveyed the fiscal stance in its member countries and expressed concern about the development of public debt in many of its member states. Projections of current fiscal policies show that in several countries debt stabilizes only far beyond the year 2000 at levels that are some 30 percentage points of GDP higher than current levels, which are already fairly high. Recent interest in the stabilization of government debt is related also to the fiscal entrance criteria of the European Monetary Union (EMU). According to the EMU provisions of the Maastricht treaty, a country can enter the EMU only if its government debt is below 60% of GDP or if the government debt–GDP ratio is approaching this target value with sufficient speed.

In the face of the dynamic government budget constraint, fiscal and monetary policy authorities typically face a conflict about whether fiscal or monetary instruments should be adjusted to stabilize government debt. Several papers have explored the interaction between fiscal and monetary policy in a game-theoretic framework. In a static context, Alesina and Tabellini (1987) include the private sector as a third player in the monetary–fiscal policy game. In a dynamic framework, Petit (1989) and Hughes Hallet and Petit (1990) consider open-loop equilibria in which the private sector plays a passive, nonstrategic role. Levine and Pearlman (1992) and Levine and Brociner (1994) consider the interaction between fiscal authorities of two countries and a single monetary authority in a monetary union. They adopt a model for private-sector behavior based on rigorous micro-foundations. Following almost the entire literature on the strategic interaction between fiscal and monetary policies in a dynamic game-theoretic framework, they have to rely on numerical simulation.

Our paper derives analytical solutions by building on the differential game framework developed by Tabellini (1986) to formalize the conflict between monetary and fiscal authorities implied by the government budget constraint. In particular, we extend Tabellini's analysis in several directions. First, as the authority that is most closely tied to the political process, the fiscal authority cares about not only fiscal but also monetary objectives. This alleviates the conflict between the two authorities by reducing the externalities that monetary policy imposes on the fiscal player. Second, in accordance with the entrance requirements for the EMU, nonzero debt targets are allowed for.

Third, and most importantly, we elaborate on the solutions derived by Tabellini (1986) in several ways. In particular, we calculate analytical solutions not only for the steady state but also for the entire transition. Moreover, we provide new interpretations of the closed-form solutions by distinguishing between the *inter*- and *intra*temporal distribution of the burden associated with government debt stabilization. We also determine and interpret the various externalities the two policy authorities impose on each other in the noncooperative game. Furthermore, the effects of changes in the objective functions of monetary and fiscal authorities are derived. In particular, we consider the effects of a more conservative central bank and reinterpret unpleasant monetarist arithmetic, which was first emphasized by Sargent and Wallace (1981), as a possible outcome of a differential debt-stabilization game between monetary and fiscal authorities. Whereas Sargent and Wallace (1981) assume that fiscal policy is exogenously given, we determine both fiscal and monetary policy endogenously as the outcome of strategic interaction between monetary and fiscal policymakers.[1] Finally, we address the issue of central bank independence by investigating Stackelberg equilibria.

The main contribution of this paper thus involves the interpretation

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1 The empirical analysis of Burdekin and Laney (1988) suggests that this two-way causality between monetary and fiscal policy is indeed important.

of analytical solutions, thereby providing insight into the contrasts between noncooperative and cooperative games. To arrive at closed-form analytical solutions for the noncooperative games and their associated inefficiencies due to externalities, we assume an open-loop information structure.[2] This implies that monetary and fiscal policies can precommit to a future course of action as departures of announced strategies would give rise to serious loss of reputation.[3] Closed-loop equilibria, which cannot be solved analytically, intensify the contrast between cooperative and noncooperative games further by exacerbating the inefficiencies in the noncooperative solution (see Tabellini, 1986). Accordingly, to analytically identify the major differences between cooperative and noncooperative solutions, we can limit ourselves to open-loop equilibria.

Section 2 introduces the differential game between fiscal and monetary authorities. Section 3 provides the analytical solutions for both the cooperative and the noncooperative Nash open-loop equilibria, which are compared in Sect. 4. By exploring changes in preference functions, Sect. 5 investigates the consequences of a more conservative monetary authority. It also interprets the possibility of "unpleasant monetarist arithmetic" in an explicit dynamic game-theoretic framework. To explore how a more independent central bank affects the strategic interactions between policymakers, Sect. 6 explores Stackelberg equilibria.

2 A Differential Game on Government Debt Stabilization

Fiscal deficits have to be financed by either base-money creation or the accumulation of government debt. In many cases, decisions on primary fiscal deficits are decentralized to the Treasury, while management of monetary policy is the responsibility of the central bank. While monetary and fiscal policies are thus delegated to different institutions, the government budget constraint renders these policies interdependent. In particular, the dynamic government budget constraint links primary fiscal deficits, $f(t)$, seignorage (or the issue of base money), $m(t)$, interest payments on government debt, $rd(t)$, and government debt accumulation $\dot{d}$ (where a dot above a variable refers to a time derivative):

$$\dot{d} = rd(t) + f(t) - m(t) . \tag{1}$$

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2 For a similar open-loop approach, see Petit (1989) and Hughes Hallett and Petit (1990). Using differential games, Blake and Westaway (1992) and Blake (1992) investigate how different assumptions about information structure and commitment affect the interaction between fiscal and monetary policies.

3 Such binding commitments are facilitated by the surveillance of national policies by the European Commission or other international organizations.

$d(t)$, $f(t)$, and $m(t)$ are expressed as fractions of GDP. r represents the rate of interest on outstanding government debt minus the growth rate of output and is assumed to be exogenous and therefore independent of the level of government indebtedness.

If the fiscal deficit, $f(t)+rd(t)$, exceeds seignorage from base-money creation, $m(t)$, government debt accumulation allows policy-makers to shift to the future the adjustment burden associated with the (nonmonetized) fiscal deficit. The dynamic government budget constraint thus reveals that the interaction between the monetary and fiscal authorities exhibits both an *intra*temporal and an *inter*temporal dimension. The latter implies a link between monetary and fiscal policies and the accumulation of government debt. The initial stock of outstanding government debt $d(0)$ and the interest rate (net of output growth) play a major role in the process of fiscal consolidation. In the presence of a large initial stock of debt and a high real interest rate, government debt stabilization requires greater efforts than called for in a situation with a low initial stock of debt and low interest rates.

Government solvency is ensured if we assume that the following transversality condition, generally referred to as the no-Ponzi game condition, is met:

$$\lim_{t\to\infty} d(t)e^{-rt} = 0 . \tag{2}$$

Stabilization of government debt can be achieved in two alternative ways: by reducing primary fiscal deficits or by raising the creation of base money. Policy conflicts arise if fiscal and monetary policies are controlled by different institutions that assign different weights to various objectives, including inflation, government debt stabilization, and public spending. Following Tabellini (1986), we formalize the strategic interaction between monetary and fiscal authorities by specifying instruments and objectives of the policymakers within a differential game.[4]

The fiscal authority (or Treasury) features the following intertemporal loss function, which depends on the time profiles of the primary fiscal deficit, base-money growth and government debt:

$$\begin{aligned} L^{\mathrm{F}}(t_0) = \frac{1}{2}\int_{t_0}^{\infty} &\big\{(f(t)-\bar{f})^2 + \eta(m(t)-\bar{m})^2 \\ &+ \lambda(d(t)-\bar{d})^2\big\} e^{-\delta(t-t_0)}\,\mathrm{d}t . \end{aligned} \tag{3}$$

4 See Basar and Olsder (1982) for details on differential game theory.

Fiscal authorities manage primary fiscal deficits to minimize this intertemporal loss function, subject to the dynamic government budget constraint (1), the transversality condition (2), and the initial stock of government debt, $d(0)$. $\bar{f}$, $\bar{m}$, and $\bar{d}$ represent exogenous policy targets for base-money growth, the primary fiscal deficit, and public debt. These "blisspoints" reflect the institutional and political structures in which decisionmaking on macroeconomic policies takes place. The subjective rate of time preference, δ, determines the extent to which policymakers discount future losses.[5]

The primary deficit reflects the objectives of the fiscal authority with respect to noninterest public spending and taxation. The policy target for the primary deficit, $\bar{f}$, can be interpreted as the blisspoint of noninterest public spending given an exogenous path for taxes. Alternatively, it can be viewed as the preferred tax–GDP ratio given an exogenous path for noninterest public spending.

As in Tabellini (1986), government debt features in the loss function because higher levels of debt require larger tax distortions to service the additional interest payments. Moreover, the larger the stock of public debt becomes, the more substantial the required adjustments in taxes associated with fluctuations in the real rate of interest and real output need to be. If Ricardian equivalence fails, high levels of public debt are likely to also crowd out private investment and induce undesirable intergenerational redistributions of wealth. In view of these arguments, Tabellini (1986) assumes a zero policy target for public debt. The entrance requirement for the EMU, however, involves a debt–GDP ratio of 60%. Moreover, several countries have implemented positive debt targets to guide the process of fiscal consolidation (see Richardson et al., 1994). Accordingly, we allow for a positive debt target to model the desire to comply with such debt targets. The parameter λ reflects the priority that the fiscal authorities attach to debt stabilization. A large weight can be interpreted as the fiscal authorities wanting to exercise substantial fiscal discipline.

As another extension of Tabellini (1986), growth of base money enters the objective function of the fiscal authorities. Money growth is included in the objective function because the fiscal authority, which is closely linked to the political process and thus represents the interests of the electorate, cares about inflation.[6]

5 A high rate of time preference is sometimes associated with a high degree of political instability.

6 We assume that the economy is on the upward-sloping part of the seignorage Laffer curve so that a higher rate of inflation increases seignorage revenues, $m(t)$. Empirical studies on money demand indicate that inflation in

Monetary authorities set the growth of base money so as to minimize the following loss function:

$$L^{\mathrm{M}}(t_0) = \frac{1}{2}\int_{t_0}^{\infty} \{(m(t) - \bar{m})^2 + \kappa(d(t) - \bar{d})^2\} e^{-\delta(t-t_0)}\,\mathrm{d}t \ . \quad (4)$$

The relative weight attached to money growth (i.e., $1/\kappa$) measures how conservative the central bank is. In particular, if $\kappa = 0$ (so that $1/\kappa \to \infty$), the central bank cares only about price stability, and thus is ultra conservative.

The instrument controlled by the fiscal player (i.e., the primary fiscal deficit) does not enter the objective function of the monetary player. The fiscal player, however, cares about the instrument controlled by the monetary player (i.e., base-money growth). This asymmetry originates in the different positions of the monetary and fiscal authorities vis-à-vis the political process. The monetary authorities are relatively independent from the political process. The fiscal authorities, in contrast, are closely linked to the government, which is responsible to parliament. As such, the fiscal player represents more closely the political objectives of the electorate, which cares not only about public spending and taxes but also about inflation.

Including money growth in the objective function of the fiscal authorities does not affect the noncooperative Nash open-loop equilibrium. The reason is that the fiscal player takes the strategy of the monetary player as given. Accordingly, it does not perceive any effect of its instrument (i.e., the primary fiscal deficit) on money growth.[7] The cooperative equilibrium and thus the externalities in the noncooperative game, however, are affected by the inclusion of money growth in the objective function of the fiscal authorities. In particular, the policy conflicts between the two authorities about money growth are alleviated. If money creation does not enter the fiscal objective function, a reduction in money growth by the central bank imposes an adverse externality on

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industrial countries is well below the seignorage maximizing rate (see, e.g., Boughton, 1991). Accordingly, industrial countries are indeed on the upward-sloping part of the seignorage Laffer curve. If money demand is of the constant velocity type, i.e., $M(t) = kP(t)y(t)$ [where $P(t)$ denotes the price level, $y(t)$ real output, and k velocity] and price expectations are rational (so that the inflation tax on government debt is zero), inflation π is given by $\pi = km(t) - g_y$, where g_y denotes real output growth.

7 If the fiscal player is Stackelberg leader, however, it takes into account how the central bank responds to changes in fiscal policy. Accordingly, it perceives an indirect effect of fiscal policy on money growth (see Sect. 6).

the fiscal authorities by boosting the accumulation of public debt. With money growth entering the loss function of the fiscal authorities, this adverse externality is partially offset by a positive externality implied by lower money growth and thus lower inflation.

The gap between $\bar{f} + r\bar{d}$ and $\bar{m}$,[8] which is assumed to be positive, is an important determinant of government debt accumulation: it measures the tension between the desired financing, $\bar{f} + r\bar{d}$, and desired monetary accommodation, $\bar{m}$. Accordingly, a larger gap intensifies the conflict between the target values for the various objectives. Also a large initial stock of government debt, $d(0)$, or a low debt target increase the tension between the policy objectives. In the remainder of the analysis, we assume that the initial stock of debt exceeds the target, i.e., $d(0) > \bar{d}$. Another important factor behind the accumulation of public debt is the difference between the rate of time preference, δ, and the net interest rate, r. If $\delta > r$ and public debt does not directly feature in the objective functions (i.e., $\lambda = \kappa = 0$), the subjective benefits of additional government debt exceed its objective costs so that government debt would accumulate without bound.

The weights that the fiscal and monetary authorities attach to debt stabilization (i.e., λ and κ, respectively) are important determinants of how the burden of stabilizing government debt is distributed over the fiscal and monetary policymakers in the noncooperative Nash game. If κ is high and λ low, substantial money creation rather than small primary fiscal deficits resolve the tension between the Treasury and the central bank on government debt stabilization. Hence, this situation implies a strong fiscal player and a weak central bank. If both κ and λ are small, neither player is willing to substantially adjust its policy to stabilize government debt. Hence, if the policy authorities are relatively impatient (i.e., $\delta > r$), the adjustment burden is shifted mainly towards the future by accumulating more public debt.

3 Solving the Differential Game

Two elements are crucial in the dynamic interaction between monetary and fiscal authorities: namely, first, whether policies are coordinated and, second, the information structure. Coordination of macroeconomic policies internalizes the positive externalities on the other player from efforts to stabilize government debt. The cooperative equilibrium is thus

8 We assume that both players feature the same blisspoints. Allowing both players to differ with respect to their targets would not qualitatively affect the analysis.

Pareto efficient and can therefore serve as a benchmark to determine the inefficiencies associated with noncooperative equilibria. In terms of institutional settings, the cooperative equilibrium can be interpreted as involving a central coordinating institution, such as parliament, determining guidelines for the time paths of fiscal deficits and base-money creation (see Tabellini, 1986). In the case of cooperation, ω represents the relative weight attached to the objectives of the fiscal authorities in the aggregate loss function of the coalition, $L^{C}(t_0) = \omega L^{F}(t_0) + L^{M}(t_0)$. ω can be interpreted as the bargaining strength of the fiscal authorities in a cooperative game with the monetary authorities or, alternatively, as the political strength of the Treasury in convincing parliament about the desirability of its preferred policies.[9]

The cooperative equilibrium is found by minimizing the following present-value Hamiltonian,

$$\begin{aligned} H^{C}(t) = {} & \frac{\omega}{2}(f(t) - \bar{f})^2 + \frac{\omega\eta + 1}{2}(m(t) - \bar{m})^2 \\ & + \frac{\omega\lambda + \kappa}{2}(d(t) - \bar{d})^2 + \mu^{C}(t)(rd(t) + f(t) - m(t)) \end{aligned} \tag{5}$$

with respect to the available instruments $\{f(t), m(t)\}$. As the co-state variable associated with the dynamic government budget constraint, $\mu^{C}(t)$ represents the marginal costs of public funds as perceived by the coalition of policymakers. The first-order conditions of this dynamic optimization problem amount to:

$$\left.\begin{aligned} f(t) &= \bar{f} - \frac{\mu^{C}(t)}{\omega}, \\ m(t) &= \bar{m} + \frac{\mu^{C}(t)}{\omega\eta + 1}, \\ \dot{\mu}^{C} &= (\delta - r)\mu^{C}(t) - (\omega\lambda + \kappa)(d(t) - \bar{d}). \end{aligned}\right\} \tag{6}$$

The noncooperative game suffers from inefficiencies. To analytically explore the inefficiencies in noncooperative equilibria, we focus on open-loop equilibria. As noted in the introduction, closed-loop equilibria, which cannot be solved analytically, would only further inten-

9 We assume ω to be exogenous. Alternatively, it could be determined endogenously as the outcome of cooperative bargaining, for example as the Nash-bargaining solution.

sify the contrast between cooperative and noncooperative games. In this section, we solve for the Nash game before turning (in Sect. 6) to Stackelberg games. In the Nash open-loop game, players simultaneously commit to a strategy, taking as given not only the current decision of the opponent but also the future course of action of the other player. In terms of institutional arrangements, the Nash open-loop equilibrium can be interpreted as the two policymakers simultaneously submitting their strategies to a third authority enforcing these plans as binding commitments. This third authority can be the European Commission or another international organization conducting surveillance of national policies.

The Nash open-loop equilibrium is found by separately maximizing the present-value Hamiltonians of the fiscal and monetary authorities. The Hamiltonian of the fiscal authorities is given by:

$$H^{\mathrm{F}}(t) = \frac{1}{2}(f(t) - \bar{f})^2 + \frac{\eta}{2}(m(t) - \bar{m})^2 + \frac{\lambda}{2}(d(t) - \bar{d})^2 \\ + \mu^{\mathrm{F}}(t)(rd(t) + f(t) - m(t)) \; . \tag{7}$$

This gives rise to the following first-order conditions:

$$\begin{aligned} f(t) &= \bar{f} - \mu^{\mathrm{F}}(t) \; , \\ \dot{\mu}^{\mathrm{F}}(t) &= (\delta - r)\mu^{\mathrm{F}}(t) - \lambda(d(t) - \bar{d}) \; . \end{aligned} \tag{8}$$

Maximization of the present-value Hamiltonian of the monetary authorities,

$$H^{\mathrm{M}}(t) = \frac{1}{2}(m(t) - \bar{m})^2 + \frac{\kappa}{2}(d(t) - \bar{d})^2 \\ + \mu^{\mathrm{M}}(t)(rd(t) + f(t) - m(t)) \tag{9}$$

yields the following optimality conditions:

$$\begin{aligned} m(t) &= \bar{m} + \mu^{\mathrm{M}}(t) \; , \\ \dot{\mu}^{\mathrm{M}} &= (\delta - r)\mu^{\mathrm{M}}(t) - \kappa(d(t) - \bar{d}) \; . \end{aligned} \tag{10}$$

The optimization of quadratic objective functions produces linear dynamic systems of government debt and the co-state variables associated with government debt, $\mu^i(t)$. These dynamic systems are assumed to display saddlepoint stability to rule out explosive government indebtedness and thus violation of the transversality constraint (2). This requires that the weights policy authorities attach to debt stabilization (i.e., λ and κ) are large relative to the gap between the subjective rate of time preference and the net interest rates (for more details, see Sect. 4 below).

In the saddlepoint stable systems, adjustments in the forward-looking co-state variables place the system on the unique converging trajectory to its new steady-state equilibrium $\{d(\infty), \mu^i(\infty)\}$. The stable root of the dynamic system in $\{d(t), \mu^i(t)\}$ determines the transient dynamics of the saddlepoint stable system: with a negative sign it measures the adjustment speed towards the steady state. The adjustment speed is denoted by h.[10]

The system dynamics of both the cooperative and noncooperative game can be written in the following form:

$$\left.\begin{aligned} d(t) &= (d(0) - d(\infty))e^{-ht} + d(\infty)\,, \\ \mu^i(t) &= (\mu^i(0) - \mu^i(\infty))e^{-ht} + \mu^i(\infty)\,, \\ f(t) &= (f(0) - f(\infty))e^{-ht} + f(\infty)\,, \\ m(t) &= (m(0) - m(\infty))e^{-ht} + m(\infty)\,, \end{aligned}\right\} \tag{11}$$

where ∞ refers to the steady state and 0 to the initial state of a variable. Expression (11) reveals that the dynamics of any variable can be characterized by three elements: the initial state, the adjustment speed, h, and the steady state. The next section derives these three elements analytically for both the cooperative and the noncooperative Nash open-loop equilibria.

4 Cooperative and Noncooperative Nash Equilibria

The cooperative equilibrium and the noncooperative Nash open-loop equilibria can be solved analytically. Table 1 provides the initial value, the adjustment speed, and the steady-state of both equilibria.[11] Table 1 implies that short-run and long-run money growth and primary fiscal deficit and long-run debt can be expressed in terms of two elements: first, a parameter indicating the *intra*temporal distribution of the adjustment burden associated with government debt stabilization, α,[12] and,

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10 The reciprocal of the adjustment speed, $1/h$, equals the mean time lag, which is defined as the time that is required to eliminate about 63% of the discrepancy between the initial and the steady-state value of a particular variable.

11 An appendix available on request from the authors contains the derivation of Table 1.

12 α and $1 - \alpha$ are closely related to the "feedback"-coefficients θ_1 and

Table 1: General solution

$$m(0) = \bar{m} + (1-\alpha)[(1-h\beta)(\bar{f} + r\bar{d} - \bar{m}) + (h+r)(d(0) - \bar{d})]$$
$$f(0) = \bar{f} - \alpha[(1-h\beta)(\bar{f} + r\bar{d} - \bar{m}) + (h+r)(d(0) - \bar{d})]$$
$$f(0) - m(0) = h\beta(\bar{f} + r\bar{d} - \bar{m}) - h(d(0) - \bar{d}) - rd(0)$$
$$m(\infty) = \bar{m} + (1-\alpha)(1+r\beta)(\bar{f} + r\bar{d} - \bar{m})$$
$$f(\infty) = \bar{f} - \alpha(1+r\beta)(\bar{f} + r\bar{d} - \bar{m})$$
$$f(\infty) - m(\infty) = -r\bar{d} - r\beta(\bar{f} + r\bar{d} - \bar{m})$$
$$d(\infty) = \bar{d} + \beta(\bar{f} + r\bar{d} - \bar{m})$$
$$f(\infty) - f(0) = -\alpha(h+r)[\beta(\bar{f} + r\bar{d} - \bar{m}) - (d(0) - \bar{d})]$$
$$m(\infty) - m(0) = (1-\alpha)(h+r)[\beta(\bar{f} + r\bar{d} - \bar{m}) - (d(0) - \bar{d})]$$
$$d(\infty) - d(0) = \beta(\bar{f} + r\bar{d} - \bar{m}) + (\bar{d} - d(0))$$
$$h = -\frac{\delta}{2} + \frac{\sqrt{\delta^2 + 4\Delta}}{2}$$
$$\beta = \frac{(\delta - r)}{\Delta} = \frac{(\delta - r)}{h(h+\delta)}$$

second, a parameter indicating the *inter*temporal distribution of that burden, β. This latter parameter is inversely related to the adjustment speed, h (which, in turn, is positively related to Δ, which denotes the negative of the determinant of the saddlepoint stable system). Both the cooperative and the noncooperative games yield the same expressions for initial and steady-state money growth, primary fiscal deficits, and steady-state debt in terms of Δ (or h or β) and α. Table 2 gives the solutions of Δ and α in the cooperative and Nash open-loop equilibrium.

The parameter α indicates how the *intra*temporal adjustment burden is distributed over the two authorities: α represents the fiscal authorities' share of the adjustment burden, while the complementary share borne by the monetary authorities is given by $1-\alpha$. A large α indicates relatively weak fiscal authorities which bear most of the adjustment burden. In particular, the primary fiscal deficit, $f(t)$, is much below its

π_1 used by Tabellini (1986). These coefficients measure by how much the monetary and fiscal authorities adjust their policies to changes in the current stock of debt. Since the game is linear-quadratic, Tabellini (1986) proposes the following feedback relations: $m(t) = \theta_0 + \theta_1 d(t)$ and $f(t) = \pi_0 - \pi_1 d(t)$. Table 1 and (11) imply the following: $\theta_0 = m(0) - (1-\alpha)(h+r)d(0)$, $\theta_1 = -\alpha(h+r)$, $\pi_0 = f(0) + \alpha(h+r)d(0)$, and $\pi_1 = (1-\alpha)(h+r)$.

Table 2: Δ and α in the cooperative equilibrium and the noncooperative Nash open-loop equilibrium

	Cooperation	Nash open-loop
Δ	$(\omega\lambda+\kappa)(\frac{1}{\omega}+\frac{1}{\omega\eta+1})-r(\delta-r)$	$\lambda+\kappa-r(\delta-r)$
α	$\frac{1/\omega}{1/\omega+1/(\omega\eta+1)}=\frac{\omega\eta+1}{\omega\eta+1+\omega}$	$\frac{\lambda}{\lambda+\kappa}$

blisspoint, $\bar{f}$, while money growth is relatively close to its target value, $\bar{m}$. If the fiscal authorities are strong and the central bank is weak (i.e., α is small), in contrast, debt stabilization is achieved mainly through monetization of fiscal deficits.

The parameter β reveals how the adjustment burden is shifted intertemporally. This parameter is zero if authorities are patient (i.e., $\delta = r$). This indicates that the conflict between fiscal and monetary policies is resolved without shifting the adjustment burden intertemporally. However, adjustment is largely shifted to the future if β is large, which occurs if policymakers are impatient (i.e., $\delta > r$) and attach a low weight to debt stabilization. In that case the adjustment speed, h, is low.[13] The impact of β on short-run and long-run policy variables reflects the *inter*temporal distribution of the adjustment burden. A higher value of β implies higher deficits and lower money growth in the short run, but lower deficits and higher money growth in the long run.[14] With a positive value of β, the tension between the various objectives, which is indicated by the difference between $\bar{f}+r\bar{d}$ and $\bar{m}$, is not fully resolved in the short run. The associated "underadjustment" results in the accumulation of government debt. The resulting additional interest payments on a higher stock of government debt $[d(\infty) > \bar{d}]$ require the authorities to "overadjust" in the long run in the sense that they have to depart more from their blisspoints than is indicated by the gap $\bar{f}+r\bar{d}-\bar{m} > 0$.

Saddlepoint stability requires Δ^{C} and Δ^{O} to be positive. The expressions in Table 2 reveal that positive values for Δ require that policymak-

13 If the weights attached to debt stabilization are low, Δ is small (see Table 2). According to Table 1, this implies that h is small as well. The speed of adjustment is thus of interest not only for its own sake but also as an indicator of intertemporal shifting. In particular, a small adjustment speed indicates that intertemporal shifting is important.

14 Recall that we assume that $\bar{f}+r\bar{d}-\bar{m} > 0$.

ers attach a sufficiently high priority to government debt stabilization (i.e., high values of λ and κ) as long as δ exceeds r.[15] Intuitively, to avoid explosive debt dynamics, authorities need to attach a sufficiently high priority to debt stabilization to offset their impatience. Adjustment is slow if the dynamic system is close to being unstable, i.e., the value of Δ is small. In particular, debt stabilization is a time-consuming process if authorities are impatient (i.e., δ exceeds r by a large margin) and at the same time care little about debt stabilization (i.e., λ and κ are small).

The expressions for α show that the intratemporal share of the adjustment burden that falls on the fiscal authorities is inversely related to ω in the cooperative case. At the same time, a higher weight of money growth (i.e., inflation) in the objective function of the fiscal authorities raises α. In the Nash open-loop case, α depends only on the relative weights attached to debt stabilization by the monetary and fiscal authorities. In particular, if the monetary authorities value debt stabilization more than the fiscal authorities do (i.e., when κ/λ is large so that the monetary authorities are not conservative, while the fiscal authorities are not disciplined), they bear most of the adjustment burden associated with the conflict between monetary and fiscal policies. As explained in Sect. 2, η does not affect the open-loop equilibrium because, in deciding on the primary fiscal deficit, the fiscal authorities take money growth as given.

A comparison of the cooperative and Nash open-loop equilibria leads to the following proposition.

Proposition 1: The speed of adjustment is higher and steady-state debt is lower in the cooperative equilibrium if either (a) fiscal authorities attach less weight to inflation stabilization relative to debt stabilization than the monetary authorities do, i.e., if $\lambda/\eta > \kappa$, or (b) fiscal authorities carry only a small weight, ω, in the cooperative case.

Proof: According to Table 1, $h^{\mathrm{C}} = -\delta/2 + \sqrt{\delta^2 + 4\Delta^{\mathrm{C}}}/2$ and $h^{\mathrm{O}} = -\delta/2 + \sqrt{\delta^2 + 4\Delta^{\mathrm{O}}}/2$. Therefore, $h^{\mathrm{C}} > h^{\mathrm{O}}$ if $\Delta^{\mathrm{C}} > \Delta^{\mathrm{O}}$. If we insert the definitions of Δ^{C} and Δ^{O} from Table 2, we arrive at the following inequalities:

15 These stability conditions are consistent with Tabellini (1986, p. 431), who stresses that the time path of debt can be stable even if $r > 0$.

$$
\begin{aligned}
&(\omega\lambda + \kappa)\left(\frac{1}{\omega} + \frac{1}{\omega\eta + 1}\right) > \lambda + \kappa \\
&\iff (\lambda - \eta\kappa)\frac{\omega}{\omega\eta + 1} + \frac{\kappa}{\omega} > 0 \, .
\end{aligned} \tag{12}
$$

The inequality $(\lambda - \eta\kappa) > 0$ is a sufficient condition for the second inequality in (12) to hold. If $\omega \downarrow 0$, the second term on the left-hand side of the second inequality (12) dominates the first term. Hence, the inequality also holds. According to the definition of β in Table 1, β and h are negatively related. A positive relationship between β and $d(\infty)$ exists according to the definition of $d(\infty)$ in Table 1. Steady-state government debt, therefore, is inversely related to the adjustment speed. Hence, $h^{\mathrm{C}} > h^{\mathrm{O}}$ implies $d^{\mathrm{C}}(\infty) < d^{\mathrm{O}}(\infty)$. □

The first term at the left-hand side of the second inequality in (12) reflects the externality on "fiscal" welfare if monetary authorities raise money growth to stabilize government debt. If the fiscal authorities do not care about money growth (i.e., $\eta = 0$), this externality is positive as more money growth reduces debt accumulation. The positive externality is alleviated by a negative externality if the fiscal authority dislikes money growth (i.e., $\eta > 0$). In the remainder of this paper, we assume that $\lambda > \eta\kappa$ so that the net effect of both externalities of money growth on fiscal welfare is positive. The second term at the left-hand side of the second inequality in (12) measures the positive spillover on "monetary" welfare if the fiscal authorities decrease the primary fiscal deficit to stabilize government debt. The cooperative equilibrium internalizes the positive spillover on the monetary authority of reductions in the primary fiscal deficit and the positive spillover on the fiscal authority associated with increases in money growth. Both actions reduce government debt accumulation, thereby lowering the steady-state stock of debt and speeding up adjustment.

The information in Tables 1 and 2 reveals that short-term fiscal policies are typically too loose, while monetary policies are too tight in the noncooperative equilibrium. In particular, $[f(0) - m(0)]$ is smaller with cooperation than without cooperation, since $h^{\mathrm{C}}\beta^{\mathrm{C}}$ is smaller[16] than $h^{\mathrm{O}}\beta^{\mathrm{O}}$ and h^{C} is larger[17] than h^{O}. The intertemporal budget constraint then implies smaller long-run debt, allowing $[f(\infty) - m(\infty)]$ to be larger under cooperation than under noncooperation (see Table 1 and recall that β^{C} is smaller than β^{O}). Regarding steady-state primary fis-

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16 Recall that $\bar{f} + r\bar{d} - \bar{m} > 0$.

17 Recall that $d(0) > \bar{d}$.

cal deficits and money creation in both equilibria, we can show the following result.

Proposition 2: Steady-state primary fiscal deficits are lower in the cooperative equilibrium than in the noncooperative Nash open-loop equilibrium, if

$$\kappa - \omega(\lambda - \kappa\eta) > \frac{r(\delta - r)\kappa}{\omega\lambda + \kappa}(\omega\eta + 1)\ . \tag{13a}$$

Steady-state money creation is lower in the cooperative equilibrium than in the noncooperative Nash open-loop equilibrium, if

$$\kappa - \omega(\lambda - \kappa\eta) > -\frac{r(\delta - r)(\lambda - \kappa\eta)}{\omega\lambda + \kappa}\omega^2\ . \tag{13b}$$

Proof: From Table 1 it follows that $f^{\mathrm{C}}(\infty) < f^{\mathrm{O}}(\infty)$ if $\alpha^{\mathrm{C}}(1+r\beta^{\mathrm{C}}) > \alpha^{\mathrm{O}}(1 + r\beta^{\mathrm{O}})$. With the definitions for α^i and Δ^i from Table 2, this inequality can be rewritten as

$$\frac{\frac{1}{\omega}}{\frac{1}{\omega} + \frac{1}{\omega\eta+1}}\left(\frac{\left(\frac{1}{\omega} + \frac{1}{\omega\eta+1}\right)(\omega\lambda + \kappa)}{\left(\frac{1}{\omega} + \frac{1}{\omega\eta+1}\right)(\omega\lambda + \kappa) - r(\delta - r)}\right) > \frac{\lambda}{\lambda + \kappa - r(\delta - r)}\ .$$

Dividing both the numerator and the denominator of the left-hand side by $\omega\lambda + \kappa$ and at the right-hand side by λ, we arrive at

$$\frac{1}{\omega\left[\left(\frac{1}{\omega} + \frac{1}{\omega\eta+1}\right) - \frac{r(\delta-r)}{\omega\lambda+\kappa}\right]} > \frac{1}{1 + \frac{\kappa}{\lambda} - \frac{r(\delta-r)}{\lambda}}\ ,$$

which implies

$$\omega\left[\left(\frac{1}{\omega} + \frac{1}{\omega\eta + 1}\right) - \frac{r(\delta - r)}{\omega\lambda + \kappa}\right] < 1 + \frac{\kappa}{\lambda} - \frac{r(\delta - r)}{\lambda}.$$

Multiplying both sides of this inequality by $(\omega\eta + 1)\lambda$ and collecting terms yields (13a).

In a similar vein, $m^{\mathrm{C}}(\infty) < m^{\mathrm{O}}(\infty)$ if $(1 - \alpha^{\mathrm{C}})(1 + r\beta^{\mathrm{C}}) > (1 - \alpha^{\mathrm{O}})(1 + r\beta^{\mathrm{O}})$. Substituting the expressions for α^i and Δ^i from Table 2, we find

$$\frac{\frac{1}{\omega\eta+1}}{\frac{1}{\omega} + \frac{1}{\omega\eta+1}}\left(\frac{\left(\frac{1}{\omega} + \frac{1}{\omega\eta+1}\right)(\omega\lambda + \kappa)}{\left(\frac{1}{\omega} + \frac{1}{\omega\eta+1}\right)(\omega\lambda + \kappa) - r(\delta - r)}\right) > \frac{\kappa}{\lambda + \kappa - r(\delta - r)}\ ,$$

which implies

$$(\omega\eta + 1)\left[\left(\frac{1}{\omega} + \frac{1}{\omega\eta + 1}\right) - \frac{r(\delta - r)}{\omega\lambda + \kappa}\right] < 1 + \frac{\lambda}{\kappa} - \frac{r(\delta - r)}{\kappa} .$$

Multiplying both sides of this inequality by $\omega\kappa$ and collecting terms yields (13b). □

If $\delta = r$, the policy conflict is resolved without intertemporal shifting and β equals 0. In that case, coordinated policies, compared to uncoordinated policies, are more disciplined in the long run (i.e., money growth and primary fiscal deficits are lower with cooperation), if and only if $\alpha^{C} > \alpha^{O}$. This is the case if the weight ω of the fiscal objectives in the cooperative solution is small. Intuitively, the coordinated equilibrium is dominated by the (monetary) player caring more about low inflation and less about large fiscal deficits.

If the central bank is not conservative (i.e., κ large), policies are likely to be more disciplined in the cooperative case.[18] Intuitively, in that case, the externality of the fiscal authorities on the monetary authorities (i.e., the "fiscal" externality) dominates the externality of the monetary authorities on the fiscal authorities (i.e., the "monetary" externality). Accordingly, the fiscal rather than the monetary authority has to conduct most of the adjustment in the coordinated equilibrium. If the central bank is rather conservative (small κ), policies are more disciplined without cooperation. The reason is that without cooperation the central bank is free to choose its own restrictive monetary policy without paying much attention to the consequences for public debt accumulation. Accordingly, policy cooperation worsens discipline in this case. In other words, in preserving discipline, a conservative central bank acts as a substitute for policy cooperation.

If policymakers are impatient (i.e., $\delta > r$), the condition for tighter fiscal policy in the coordinated case becomes stronger than $\alpha^{C} > \alpha^{O}$. The condition for tighter monetary policies in the case of policy cooperation, in contrast, weakens. The reason is that coordinated policies result in less accumulation of public debt. The associated lower long-run adjustment burden allows larger steady-state deficits and lower steady-state money growth. If the weight of the fiscal authorities is the same in the cooperative and Nash open-loop equilibria (i.e., $\alpha^{C} = \alpha^{O}$), cooperation implies smaller primary fiscal deficits in the short run and larger primary fiscal deficits in the long run. Intuitively, cooperation implies

18 We continue to assume here that intertemporal shifting is absent.

that the adjustment burden is shifted less to the future, as the authorities value more policies reducing debt accumulation. Accordingly, money growth is higher in the short run but lower in the long run.

5 Comparative Dynamics

The preceding section explored the features of the initial state, adjustment speed, and steady state in the cooperative and Nash open-loop equilibria. This section examines how both equilibria are affected by changes in the preference parameters and the initial stock of government debt. The effects are found by taking the partial derivatives from the expressions found in Tables 1 and 2.[19] Table 3 provides the effects of parameter changes on the initial state and the adjustment speed, h. The partial derivatives are evaluated under the assumptions that $\delta > r$, $d(0) > \bar{d}$, $\bar{f} + r\bar{d} - \bar{m} > 0$, and $\lambda - \eta\kappa > 0$. Less conservative monetary authorities (i.e., an increase of κ) raise money growth in the initial state of both equilibria and speed up the adjustment. More disciplined fiscal authorities (i.e., an increase of λ) exert similar effects. In particular, the initial primary fiscal deficit is reduced while the adjustment speed increases. In the Nash open-loop equilibrium, the sign of the effects of an increase of κ on the initial fiscal deficit and of an increase in λ on initial money creation, is ambiguous. With cooperation, in contrast, an increase in κ reduces $f^C(0)$ unambiguously and an increase in λ unambiguously raises $m^C(0)$. Intuitively, by raising the positive

Table 3: Effects of parameter changes on the initial state and the adjustment speed

	$m^C(0)$	$f^C(0)$	h^C	$m^O(0)$	$f^O(0)$	h^O
λ	+	−	+	?	−	+
κ	+	−	+	+	?	+
η	−	?	−	0	0	0
ω	+	+	+	0	0	0
$\bar{m}$	+	+	0	+	+	0
$\bar{f}$	+	+	0	+	+	0
$\bar{d}$	−	+	0	−	+	0
$d(0)$	+	−	0	+	−	0

19 An appendix containing the partial derivatives evaluated in Tables 3 and 4 is available from the authors upon request.

Table 4: Effects of parameter changes on the steady state

	$d^C(\infty)$	$m^C(\infty)$	$f^C(\infty)$	$d^O(\infty)$	$m^O(\infty)$	$f^O(\infty)$
λ	− (0)	− (0)	+ (0)	− (0)	−	? (−)
κ	− (0)	− (0)	+ (0)	− (0)	? (+)	+
η	+	?	−	0	0	0
ω	−	−	+	0	0	0
$\bar{m}$	− (0)	? (+)	+	− (0)	? (+)	+
$\bar{f}$	+ (0)	+	? (+)	+ (0)	+	? (+)
$\bar{d}$	+	+	−	+	+	−

externalities from debt stabilization in the cooperative equilibrium, a higher priority to debt stabilization attached by one player induces also the other player to be more diligent in stabilizing debt. In the noncooperative case, in contrast, the opposite reaction is likely; more efforts by one player to stabilize government debt induce the other player to reduce the efforts aimed at debt stabilization.

If the fiscal authority attaches a higher weight to monetary stability, initial money growth is lower and the adjustment speed is reduced in case of cooperation. The impact on the initial primary fiscal deficit, however, is ambiguous. A higher weight of the fiscal preferences in the cooperative game raises short-run primary fiscal deficits, money growth, and the adjustment speed. In both equilibria, a higher target for the primary fiscal deficit raises the initial primary fiscal deficit, while a lower money growth target reduces initial money growth. A higher debt target allows for lower initial money growth and a higher initial primary fiscal deficit. A higher initial stock of government debt imposes a higher initial adjustment burden and thus exerts opposite effects.

The steady-state effects of changes in the preference parameters are found in Table 4. If the signs of the partial derivatives in case $\delta = r$ differ from those in case $\delta > r$, we indicated these signs in Table 4 in parentheses. We first discuss the case of $\delta > r$. Not surprisingly, more priority attached to debt stabilization (i.e., increases in κ and λ) reduces steady-state debt. In the cooperative case, the lower debt service associated with increases in κ and λ allow for lower steady-state money growth and higher steady-state primary fiscal deficits. In the Nash open-loop case, in contrast, the effect of an increase of κ on steady-state money growth and the effect of an increase in λ on steady-state primary deficits are ambiguous. An explanation for this ambiguity is provided in Proposition 4 (see below). With policy coordination, an increase in the weight attached by the fiscal authorities to monetary stability reduces steady-state primary fiscal deficits but exerts an ambiguous impact on steady-state money growth as steady-state debt increases. More influence of the fiscal authority on cooperative policy design yields lower steady-state debt, higher steady-state primary fiscal deficits, and lower steady-state money creation.

Furthermore, a higher primary fiscal deficit target, $\bar{f}$, or a lower monetary target, $\bar{m}$, increase steady-state government debt by increasing the conflict between the various objectives. The impact of a change in the money growth target on steady-state money growth is ambiguous. The same holds true for the effect of a higher primary fiscal deficit target on steady-state primary fiscal deficits. We explore these ambiguities in more detail in Proposition 3 below. A higher debt target induces, in both equilibria, a higher steady-state level of government debt. The associated additional debt service requires higher money growth and lower primary fiscal deficits in the long run.

If $\delta = r$ (see the signs in parentheses in Table 4), the conflict between monetary and fiscal authorities is resolved without accumulation of government debt. In that case, steady-state debt is not affected by changes in the preference parameters (except for the debt target itself). In addition, ambiguities of steady-state effects on the primary fiscal deficit and money growth disappear. In particular, in the open-loop equilibrium, an increase in κ increases steady-state money growth. Furthermore, an increase in λ decreases steady-state primary fiscal deficits. Moreover, in the Nash open-loop equilibrium an increase in the primary fiscal deficit target and the money growth target increase, respectively, steady-state primary fiscal deficits and steady-state money growth in the Nash open-loop equilibrium.

The intertemporal shifting of the adjustment burden of government debt stabilization can give rise to unpleasant monetarist arithmetic of the type introduced by Sargent and Wallace (1981).[20] Unpleasant monetarist arithmetic occurs when disinflationary monetary policies have to be reversed because of higher debt accumulation that such policies induce. If fiscal authorities do not cut primary fiscal deficits to reduce government debt accumulation, i.e., if the fiscal player is strong, a large part of the debt adjustment burden is eventually shifted back to the monetary authority, resulting in higher inflation in the long run. Conversely, fiscal expansions, e.g., in the form of tax cuts, have to be reversed in the long run if the fiscal player is weak compared to the

20 Following the literature, we adopt the terminology of "unpleasant" arithmetic, without necessarily attaching a normative significance to this phenomenon.

monetary player. The adjustment burden from larger government debt that such policies produce is in the long run for the most part shifted back to the fiscal authorities, if the central bank does not monetize the additional debt. We can formulate the following proposition.

Proposition 3: A lower target value for money growth reduces money growth in the short run. However, it raises long-run money growth if (a) $\lambda < r(\delta - r) - \kappa/\omega$ in the cooperative equilibrium and (b) $\lambda < r(\delta - r)$ in the Nash open-loop equilibrium. A higher target value for primary fiscal deficits raises short-run deficits. However, it reduces long-run primary fiscal deficits if (a) $\kappa < r(\delta - r)(\omega\eta + 1) - \omega\lambda$ in the cooperative equilibrium and (b) $\kappa < r(\delta - r)$ in the Nash open-loop equilibrium.

Proof: The partial derivatives of $m(\infty)$ and $f(\infty)$ with respect to $\bar{m}$ and $\bar{f}$ are equal to

$$\frac{\partial m(\infty)}{\partial \bar{m}} = 1 - (1-\alpha)(1+r\beta) \quad \text{and} \quad \frac{\partial f(\infty)}{\partial \bar{f}} = 1 - \alpha(1+r\beta)\,,$$

respectively. A decrease of $\bar{m}$ induces an instantaneous drop in $m(0)$ in both the cooperative and Nash open-loop equilibrium since $1 - (1-\alpha^{\mathrm{C}})(1-h^{\mathrm{C}}\beta^{\mathrm{C}})$ and $1 - (1-\alpha^{\mathrm{O}})(1-h^{\mathrm{O}}\beta^{\mathrm{O}})$ are both positive. The partial derivatives of $m^{\mathrm{C}}(\infty)$ and $m^{\mathrm{O}}(\infty)$ with respect to $\bar{m}$ imply that a decrease in $\bar{m}$ induces an increase in $m(\infty)$ if (i) $1 - (1-\alpha^{\mathrm{C}})(1+r\beta^{\mathrm{C}}) < 0$ in the cooperative equilibrium, and (ii) $1 - (1-\alpha^{\mathrm{O}})(1+r\beta^{\mathrm{O}}) < 0$ in the Nash open-loop equilibrium. With the definitions of α and β in Tables 1 and 2, we can rewrite (i) as $1 - \left(\frac{1/(\omega\eta+1)}{1/\omega+1/(\omega\eta+1)}\right)\left(\frac{(\omega\lambda+\kappa)(1/\omega+1/(\omega\eta+1))}{\Delta^{\mathrm{C}}}\right) < 0$ and (ii) as $1 - \frac{\kappa}{\lambda+\kappa}\frac{\lambda+\kappa}{\Delta^{\mathrm{O}}} < 0$. Rewriting both inequalities and using the definitions of Δ^{C} and Δ^{O} from Table 2, we find in case of (i) $\frac{\lambda+\kappa/\omega-r(\delta-r)}{\Delta^{\mathrm{C}}} < 0$, whereas (ii) can be rewritten as $\frac{\lambda-r(\delta-r)}{\Delta^{\mathrm{O}}} < 0$. The first part of the proposition then follows. In a similar vein, we find that an increase in $\bar{f}$ induces an instantaneous increase in $f(0)$ in both equilibria, since $(1-\alpha^{\mathrm{C}})(1-h^{\mathrm{C}}\beta^{\mathrm{C}})$ and $(1-\alpha^{\mathrm{O}})(1-h^{\mathrm{O}}\beta^{\mathrm{O}})$ are positive. According to the partial derivatives of $f^{\mathrm{C}}(\infty)$ and $f^{\mathrm{O}}(\infty)$ w. r. t. $\bar{f}$, an increase in $\bar{f}$ causes a permanent decrease in $f(\infty)$ if (i) $1 - \left(\frac{1/\omega}{1/\omega+1/(\omega\eta+1)}\right)\left(\frac{(\omega\lambda+\kappa)(1/\omega+1/(\omega\eta+1))}{\Delta^{\mathrm{C}}}\right) < 0$ in the cooperative equilibrium and (ii) $1 - \frac{\lambda}{\lambda+\kappa}\frac{\lambda+\kappa}{\Delta^{\mathrm{O}}} < 0$ in the Nash open-loop equilibrium. (i) and (ii) can be rewritten as $\frac{((\omega\lambda+\kappa)/(\omega\eta+1))-r(\delta-r)}{\Delta^{C}}$ < 0 and $\frac{\kappa-r(\delta-r)}{\Delta^{O}} < 0$ respectively, from which the second part of the proposition follows directly. □

Unpleasant monetarist arithmetic implies that the initial disinflation is not sustainable in the long run and that the new steady state is instead characterized by a higher rate of inflation. This becomes more likely if the fiscal authorities are strong, i.e., λ is small, impatience is high, i.e., if δ is much larger than r, and if, in case of cooperation, the weight of the fiscal objective function is large, i.e., if ω is large. Unpleasant fiscal arithmetic occurs if monetary authorities are strong, i.e., κ is small, impatience is high, and if, with cooperative decision making, inflation aversion of the fiscal authority is substantial, i.e., if η is large.

Unpleasant monetarist and fiscal arithmetic can occur only if the discount rate substantially exceeds the interest rate. Indeed, a larger fiscal blisspoint unambiguously raises the long-run primary fiscal deficit if authorities are patient (i.e., $\delta = r$). If authorities are impatient, in contrast, the long-run deficit may decline if the weight attached to debt stabilization in the objective function of monetary authorities is sufficiently small.[21] Intuitively, the burden of adjustment associated with larger short-run primary deficits is not met through monetization. Instead, it is shifted to the future through debt accumulation. With the monetary authority being strong, the burden is eventually paid by the fiscal authorities in terms of lower primary deficits. Indeed, the expression for the long-run primary deficit in Table 1 reveals that unpleasant fiscal arithmetic occurs only if both β and α are large [i.e., $\alpha(1 + r\beta) > 1$]: a high value of β indicates substantial intertemporal shifting, while a high value of α indicates a strong position of the monetary authorities. Given our assumption that monetary authorities care more about inflation (relative to debt stabilization) than do fiscal authorities, i.e., that $\lambda/\eta > \kappa$, unpleasant fiscal arithmetic is less likely to occur with cooperation: the faster adjustment implies less debt accumulation as compared to the Nash open-loop equilibrium (see Proposition 1).

The issue of conservativeness of the central bank has encountered a lot of interest (see, e.g., Rogoff, 1985; Cukierman, 1992). In our

21 Only the debt stabilization weight of the monetary player appears in the condition for unpleasant fiscal arithmetic. Intuitively, unpleasant fiscal arithmetic originates in the unwillingness of the monetary player to adjust, i.e., a small value of κ. Analogously, only the priority given to debt stabilization by the fiscal player enters the condition for unpleasant monetarist arithmetic.

framework, a more conservative central bank, as measured by a lower value of κ, implies that the fiscal authorities face a larger adjustment burden from debt stabilization. This yields the following proposition.

Proposition 4: A more conservative central bank reduces the adjustment speed and raises steady-state debt. In the cooperative equilibrium (if $\delta > r$), a more conservative central bank reduces steady-state primary fiscal deficits but raises steady-state money growth. In the open-loop case, in contrast, a more conservative central bank reduces both steady-state primary fiscal deficits and money growth if $\lambda > r(\delta - r)$, i.e., if fiscal authorities are not too strong.

Proof: According to the definitions in Tables 1 and 2, a decrease in κ implies that the adjustment speed of the cooperative equilibrium and the Nash open-loop equilibrium, h^{C} and h^{O}, decrease. Since steady-state debt is negatively related to the adjustment speed (cf. Table 1), a decrease of κ increases steady-state debt [given our assumptions that $(\bar{f} + r\bar{d} - \bar{m}) > 0$ and $\delta > r$]. This proves the first part of the proposition.

In the cooperative equilibrium, κ does not affect α^{C} according to Table 2. Hence, changes in κ affect $f^{\mathrm{C}}(\infty)$ and $m^{\mathrm{C}}(\infty)$ only by changing β^{C}. If $\delta = r$, β^{C} equals 0 and changes of κ exert no effect at all. If $\delta > r$, a decrease in κ reduces the adjustment speed, implying an increase in β^{C}. A higher β^{C} implies higher steady-state government debt and, consequently, lower steady-state primary fiscal deficits and higher steady-state money growth, as stated in the middle part of the proposition.

In the Nash open-loop equilibrium, a decrease in κ increases both α^{O} and β^{O}. According to the definition in Table 1, the long-run primary fiscal deficit, $f^{\mathrm{O}}(\infty)$, decreases. Long-run money growth, $m^{\mathrm{O}}(\infty)$, decreases if κ is lowered, as long as the decrease in $(1 - \alpha^{\mathrm{O}})$ exceeds the increase in $(1 + r\beta^{\mathrm{O}})$. A decrease of κ reduces $(1 - \alpha^{\mathrm{O}})$ by and $\frac{\lambda}{(\lambda+\kappa)^2}$ leads to an increase in $(1 + r\beta^{\mathrm{O}})$ of $\frac{r(\delta-r)}{(\lambda+\kappa)^2}$. The first effect dominates the second effect if $\frac{\lambda}{(\lambda+\kappa)^2} - \frac{r(\delta-r)}{(\lambda+\kappa)^2} > 0$. This results in the last part of the proposition. □

Without cooperation, a more conservative central bank may be counterproductive in reducing long-run inflation if authorities are impatient and at the same time the fiscal authorities are strong (in the sense that they attach a low priority to stabilizing debt).[22] Hence, making a central bank more conservative is not enough to ensure low inflation rates. Low inflation rates are sustainable only if fiscal authorities are disciplined (i.e., they attach substantial weight to debt stabilization) and patient (i.e., δ does not exceed r by a large margin). This suggests that a conservative central bank needs to be complemented by fiscal reforms to achieve long-run price stability. Whereas a more conservative central bank may raise inflation in the long run, it succeeds in strengthening fiscal discipline by reducing fiscal deficits in the long run.[23]

In the presence of cooperation, a more conservative central bank raises the intertemporal shifting of the adjustment burden to the future because it reduces the overall concern about debt accumulation. This allows larger fiscal deficits and lower money growth in the short run, but requires lower deficits and higher money growth in the long run.

6 Stackelberg Equilibria of the Debt Stabilization Game

Central bank independence is another issue that has received much attention in recent years. Several countries have granted, or are intending to grant, their central banks more independence from political authorities. Furthermore, the independence of the European Central Bank (ECB) is an important part of the Maastricht treaty on the EMU. At the same time, this treaty requires that the EMU members make their national banks more independent.

This section investigates the impact of central bank independence on government debt stabilization[24] by comparing the Nash open-loop equilibrium analyzed above with the open-loop equilibria in which either the central bank or the fiscal authorities act as Stackelberg leader. The Stackelberg leader has a first-mover advantage when selecting optimal policies, and thus takes into account the response of the follower(s). The first-mover advantage gives the leader a strategic advantage in the debt stabilization game. Accordingly, on the scale of central bank in-

22 The condition for a more conservative central bank to raise long-run money growth coincides with the condition for a lower target value for money growth to raise long-run money growth in the Nash open-loop case. If the monetary authorities do not sufficiently care about debt stabilization, the model may become unstable (i.e., if $\Delta^O < 0$).

23 Only with positive public assets in the initial equilibrium [i.e., $d(0)$ negative] may fiscal policy become more expansionary in the short run.

24 See Cotarelli (1993) for a detailed survey on seignorage, central bank credit to the government, and central bank independence in practice.

Table 5: Δ and α in the two Stackelberg open-loop equilibria

	Monetary authority Stackelberg leader	Fiscal authority Stackelberg leader
Δ	$\frac{\lambda(\lambda - r(\delta - r))}{\lambda - r(\delta - r)} - \frac{r(\delta - r)(\lambda + \kappa - r(\delta - r))}{\lambda - r(\delta - r)}$	$\frac{\kappa(\kappa - r(\delta - r)) + \eta\kappa^2}{\kappa - r(\delta - r)} - \frac{r(\delta - r)(\lambda + \kappa - r(\delta - r))}{\kappa - r(\delta - r)}$
α	$1 + \frac{\kappa r(\delta - r)}{\lambda(\lambda - r(\delta - r)) - \kappa r(\delta - r)}$	$\frac{\eta\kappa^2 - \lambda r(\delta - r)}{\kappa(\kappa - r(\delta - r)) + \kappa^2\eta - \lambda r(\delta - r)}$

dependence, fiscal leadership represents the case of a dependent central bank, the Nash equilibrium is an intermediate case, while monetary leadership corresponds to independent monetary authorities.

In the context of the EMU, we can provide an alternative interpretation for Stackelberg leadership of the monetary authorities. In particular, whereas each national central bank confronts a single fiscal authority, the ECB deals with several decentralized fiscal authorities. This is likely to strengthen the strategic position of the ECB compared to that of the national central banks. Accordingly, comparing the Nash equilibrium (representing national decision making on monetary policy) and the Stackelberg equilibrium with monetary leadership (representing monetary policy set on the European level), we can explore the impact of moving monetary decision making from the national to the supranational level within the EMU.

With Stackelberg leadership, the general formulations of the solutions provided in the first part of Table 1 remain valid.[25] The values of α, which measures the intratemporal adjustment burden of the fiscal player, and Δ are contained in Table 5. As in the Nash equilibrium, the relationship between Δ and the measure of intertemporal shifting of the adjustment burden, β, is given by $\beta = (\delta - r)/\Delta$. Stability requires that $\lambda(\lambda - r(\delta - r)) - r(\delta - r)(\lambda + \kappa - r(\delta - r)) > 0$ in case the monetary authorities are Stackelberg leader, and $\kappa(\kappa - r(\delta - r)) + \kappa^2\eta - r(\delta - r)(\lambda + \kappa - r(\delta - r)) > 0$ in case the fiscal authorities are Stackelberg leader. These conditions are met if intertemporal shifting

25 Compared to the Nash equilibrium, however, the Stackelberg equilibria yield different expressions for the adjustment speed and the relationship between the adjustment speed and the indicator for intertemporal shifting, β. An appendix with the derivation of the Stackelberg solutions is available on request.

is absent (i.e., $\delta = r$) and the debt weights λ and κ are positive. In the presence of intertemporal shifting, however, stability requires that the follower attaches a sufficient weight to debt stabilization. Intuitively, the stronger strategic position allows the Stackelberg leader to shift the burden of stabilizing debt towards the follower. This results in explosive behavior of debt if this follower is not willing to adjust due to a low priority for debt stabilization.

The ability of the leader to largely avoid adjustment makes the stability conditions in the Stackelberg game more stringent than in the Nash equilibrium. To illustrate, with monetary leadership, the stability condition in the Stackelberg game excludes unpleasant monetarist arithmetic in the Nash equilibrium.[26] Hence, if a more conservative central bank is counterproductive in achieving long-run price stability, a more independent central bank produces explosive debt behavior and is thus counterproductive as well. Intuitively, making the central bank "stronger" by making it more conservative or more independent fails to achieve long-run price stability if the fiscal authorities are undisciplined and impatient. This may explain why, in the Maastricht treaty for EMU, a conservative ECB is combined with strict debt ceilings and surveillance of national fiscal policies.

With fiscal leadership, the equilibrium can be stable even if unpleasant fiscal arithmetic is present in the Nash open-loop game, i.e., if $\kappa - r(\delta - r) < 0$. The reason is that the fiscal player cares about money growth, i.e., $\eta > 0$. Hence, it will not push all the adjustment unto the central bank. Nevertheless, stability requires that the central bank is not too conservative (i.e., cares little about debt stabilization, as indicated by a small value of κ), especially if the fiscal authorities care only little about money growth.

The Stackelberg game with fiscal leadership is the only open-loop equilibrium in which the weight the fiscal authority attaches to money growth, η, affects the equilibrium. This is because, as Stackelberg leader, the fiscal player takes into account how the central bank alters its monetary policy in response to fiscal policy. Hence, through the response of monetary policy, the government perceives an indirect effect of its fiscal policy on money growth.

The intratemporal adjustment parameter, α, reflects the stronger strategic position of the Stackelberg leader. The case without intertem-

26 Stability of the Nash open-loop equilibrium requires $\lambda+\kappa-r(\delta-r) > 0$ (see Table 2), while unpleasant monetarist arithmetic requires $\lambda - r(\delta - r) < 0$ (see Proposition 3). These two inequalities imply that $\lambda(\lambda - r(\delta - r)) - r \times (\delta - r)(\lambda + \kappa - r(\delta - r)) < 0$, thereby violating the stability condition of the Stackelberg open-loop equilibrium with monetary leadership.

poral shifting (i.e., $\delta = r$) and monetary leadership illustrates this. In that case, the fiscal authorities' share of the adjustment burden is one (i.e., $\alpha = 1$), indicating that, as Stackelberg leader, the central bank can shift the entire adjustment burden to the follower. Indeed, the central bank is at its blisspoint in the long run [i.e., $m(t) = \bar{m}$ and $d(t) = \bar{d}$].

With fiscal leadership and in the absence of intertemporal shifting, the entire burden falls on the monetary authorities (i.e., $\alpha = 0$) only if the fiscal authorities do not care about money growth at all (i.e., $\eta = 0$). With $\eta > 0$, fiscal leadership yields less extreme results than monetary leadership. However, the monetary authorities' adjustment burden under fiscal leadership exceeds that in the Nash equilibrium because the fiscal authority (i.e., the Stackelberg leader) cares less about money growth (compared to debt stabilization) than do the monetary authorities (i.e., the Stackelberg follower), i.e., $\eta < \lambda/\kappa$. Accordingly, the adjustment burden of the central bank (i.e., $1-\alpha$) – and thus money growth (and inflation) – is lowest under monetary leadership and highest under fiscal leadership, with the Nash equilibrium as an intermediate case. If policy authorities are patient (i.e., $\delta = r$),[27] a more independent central bank thus strengthens policy discipline by reducing both money growth and fiscal deficits.

In the presence of intertemporal shifting (i.e., $\delta > r$), the fiscal player absorbs more than 100% of the adjustment burden if it acts as a Stackelberg follower (i.e., $\alpha > 1$). The reason is that the monetary player behaves strategically by reducing money growth below its blisspoint. By marginally decreasing money growth from its blisspoint, the central bank forces the fiscal player to intensify its efforts to reduce public debt. This yields a first-order gain in central-bank welfare by bringing debt closer to its target [as $d(\infty) > \bar{d}$ if $\delta > r$). The loss in welfare due to the decline in money growth away from the blisspoint is only second order. This yields the following proposition.

Proposition 5: Among open-loop equilibria, steady-state money growth and primary fiscal deficits are lowest with Stackelberg leadership of the central bank and (if $\lambda \geq \eta\kappa$) highest with Stackelberg leadership of the fiscal authorities. The Nash equilibrium coincides with the Stackelberg equilibrium with the central bank as leader only if $\kappa = 0$. It coincides with the Stackelberg equilibrium with the fiscal authorities as leader only if $\lambda = \eta\kappa$.

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27 In the absence of intertemporal shifting, the stability conditions are met as long as λ and κ are positive.

Proof: From Table 1, it follows that $m^{\mathrm{M}}(\infty) < m^{\mathrm{O}}(\infty)$ if $(1-\alpha^{\mathrm{M}}) \times (1+r\beta^{\mathrm{M}}) < (1-\alpha^{\mathrm{O}})(1+r\beta^{\mathrm{O}})$. Using the definitions in Tables 2 and 5, this equality can be reduced to $\kappa\lambda(\lambda - r(\delta - r)) > 0$. Stability requires $\lambda > r(\delta - r)$ and $\lambda > 0$ (see footnote 26). Hence, $m^{\mathrm{M}}(\infty) = m^{\mathrm{O}}(\infty)$ requires $\kappa = 0$. Similarly, $m^{\mathrm{O}}(\infty) < m^{\mathrm{F}}(\infty)$ if $(1-\alpha^{\mathrm{O}})(1+r\beta^{\mathrm{O}}) < (1-\alpha^{\mathrm{F}})(1+r\beta^{\mathrm{F}})$, which holds if $\kappa^2(\lambda - \eta\kappa) > 0$. Stability requires that $\kappa > 0$. Hence, $m^{\mathrm{O}}(\infty) = m^{\mathrm{F}}(\infty)$ requires that $\lambda = \eta\kappa$. $f^{\mathrm{M}}(\infty) < f^{\mathrm{O}}(\infty)$ requires that $\alpha^{\mathrm{M}}(1+r\beta^{\mathrm{M}}) > \alpha^{\mathrm{O}}(1+r\beta^{\mathrm{O}})$. This condition reduces to $\lambda^2\kappa > 0$. Finally, $f^{\mathrm{O}}(\infty) < f^{\mathrm{F}}(\infty)$ if $\alpha^{\mathrm{O}}(1+r\beta^{\mathrm{O}}) > \alpha^{\mathrm{F}}(1+r\beta^{\mathrm{F}})$ which reduces to $\kappa(\lambda - \eta\kappa)(\kappa - r(\delta - r)) > 0$. □

The Stackelberg leader undertakes little effort to stabilize government debt. Instead, it uses its policy instrument strategically to force more adjustment unto the other player, i.e., the follower. This feature makes this equilibrium particularly inefficient as reflected in substantial government debt accumulation. The following proposition can be formulated.

Proposition 6: In open-loop games, steady-state debt is higher under Stackelberg leadership than in the Nash equilibrium if, compared to the follower, the leader cares more about debt stabilization than about the instrument controlled by the follower.

Proof: First we solve $d^{\mathrm{M}}(\infty) > d^{\mathrm{O}}(\infty)$. From Table 1 and $\beta = (\delta - r)/\Delta$, this inequality holds if $\Delta^{\mathrm{M}} < \Delta^{\mathrm{O}}$. Using the expressions in Tables 2 and 5 this condition can be written as $\lambda(\lambda - r(\delta - r)) - r(\delta - r)[\lambda + \kappa - r(\delta - r)] > (\lambda - r(\delta - r))[\lambda + \kappa - r(\delta - r)]$ which holds if $\kappa\lambda > 0$. Stability requires $\lambda > 0$. Hence, the inequality is met if $\kappa > 0$, which implies that, compared to the fiscal authority, the central bank cares more about debt stabilization than about fiscal deficits (since the central bank does not care about fiscal deficits at all). Next, we solve $d^{\mathrm{F}}(\infty) > d^{\mathrm{O}}(\infty)$. $\Delta^{\mathrm{F}} < \Delta^{\mathrm{O}}$ holds if: $\kappa(\kappa - r(\delta - r)) + \kappa^2\eta - r(\delta - r)[\lambda + \kappa - r(\delta - r)] > (\kappa - r(\delta - r))[\lambda + \kappa - r(\delta - r)]$, which can be reduced to $\kappa(\lambda - \eta\kappa) > 0$. Stability requires $\kappa > 0$. Hence, the inequality is met if the priority that the fiscal authority attaches to debt stabilization relative to money growth, λ/η, exceeds the corresponding relative priority of the central bank, κ. □

This proposition implies that the Stackelberg equilibrium with the central bank as leader yields higher steady-state debt compared to the Nash equilibrium because the central bank does not care about the instrument controlled by the fiscal authorities, i.e., primary fiscal deficits.

Compared to the Nash game, the Stackelberg game with fiscal leadership yields also higher steady-state debt because the fiscal player, although caring about money growth (i.e., $\eta > 0$), attaches a higher relative priority to debt stabilization than does the monetary player (i.e., $\lambda/\eta > \kappa$).

We can conclude that, starting from an initial situation with a rather dependent central bank (i.e., fiscal Stackelberg leadership), making the central bank less dependent increases discipline on all fronts in the steady state: money growth, primary fiscal deficits, and debt all decline. An even more independent central bank (i.e., moving from a Nash equilibrium to the Stackelberg equilibrium with monetary leadership) reduces steady-state money growth and primary fiscal deficits even more. However, public debt increases. This may explain the fear that moving towards an EMU (where the monetary authorities have a stronger strategic position than with national decision making) results in excessive debt accumulation.

7 Conclusions

This paper has extended the analysis of Tabellini (1986) on the strategic interaction between monetary and fiscal authorities implied by the dynamic government budget constraint. In particular, we derived and interpreted closed-form solutions for the dynamics of fiscal deficit, money growth, and government debt in the cooperative and the Nash open-loop equilibria. Cooperation among the authorities internalizes the positive spillovers from debt stabilization efforts and thus results in faster adjustment and lower steady-state debt. It may, however, boost steady-state inflation if the central bank is weak. Among the noncooperative equilibria Stackelberg equilibria produced the largest stocks of steady-state debt as the Stackelberg leader exploits its strategic advantage to avoid adjustment.

The unpleasant monetarist arithmetic was reformulated in a dynamic game-theoretic framework in which both fiscal and monetary policy are determined endogenously. We found that a strong fiscal player – in the sense of not taking much responsibility for debt stabilization – yields unpleasant monetarist arithmetic. Moreover, we explored the impact of a more conservative central bank. In the cooperative equilibrium, a more conservative central bank boosts long-run inflation by reducing the aggregate concern for debt stabilization. In the Nash open-loop equilibrium, in contrast, a more conservative central bank reduces long-run inflation as long as the fiscal player cares sufficiently about debt stabilization and policymakers are relatively patient. Hence, to reduce long-run inflation, one may want either policy cooperation or a conservative central bank but not both together. Moreover, as an instrument to foster long-term price stability under decentralized policymaking, a more conservative central bank needs to be complemented by patient and disciplined fiscal authorities to prevent debt accumulation from undermining price stability. The analysis of Stackelberg equilibria revealed that the same holds true for a more independent central bank. These results provide a case for establishing ceilings on public debt and surveillance of national fiscal policy to help an independent, conservative European Central Bank achieve long-run price stability in the EMU.

Acknowledgements

We are indebted to two anonymous referees, and to Jacob Engwerda, Rudy Douven, and Harald Uhlig for helpful and stimulating comments.

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